

Random Vibration (Stress Screening) of Printed Wiring Assemblies

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This paper summarizes the results of a Random Vibration Stress Screening (RVSS) study on the determination of upper and lower vibration limits on Printed Wiring Assemblies (PWA). It is intended to serve as a guideline for engineers and designers who make decisions on PWA features to withstand the stresses of dynamic testing and screening. The maximum allowable PWA deflection, "G" levels, and PSD levels are compared to the expected or actual levels to determine if deleterious effects will occur.

INTRODUCTION

Analytical methods are developed to determine the maximum random vibration levels which will cause fatigue failure in the PWA components or in the bare board itself. The reversing stresses on the component leads is given special emphasis in the analysis. In the case of RVSS, the designer must attempt to determine the maximum vibration levels which can be exerted on the PWA in order to surface the maximum number of quality and workmanship defects without causing deleterious effects to the components or the board. Often RVSS levels are chosen as a default from a DOD guideline and the subsequent screen is either benign or overstressful. Thus it is useful to have design or test guidance on the upper vibration limits for the PWA. The lower limit for RVSS is needed to determine if there will be sufficient force in

the RVSS to surface a sufficient number of available workmanship or quality defects in the PWA.

Random vibration is basically a gaussian distribution of reversing force vectors and as such, it is necessary to make some simplifying assumptions in the development of the algorithms presented here. The simplifications are reasonable first estimates; however, they should be considered estimates and subject to test verification. Only classical cases were analyzed since there is an extremely large number of possible configurations and variations of PWA's and component mix. The components which were analyzed are dual-in-line micro electronic devices (IC's), capacitors, resistors, and leadless chip carriers (LCC's).

TYPICAL RVSS PLAN

The plans shown in Figures 1(a) - 1(c) indicate the basic process steps and the relative reliability (\bar{R}) of the process. The indicated reliability is only used to illustrate the relative degree of improvement and not a measured value. Plan 1(a) shows only a one pass RVSS and a test with a repair process. A repaired PWA is not re-screened. Process 1(b) allows for re-screening to filter out any defective component or workmanship flaws. Process 1(c) basically is the same as 1b except that a small PWA test is added during RVSS to detect any intermittents which may occur with vibration.

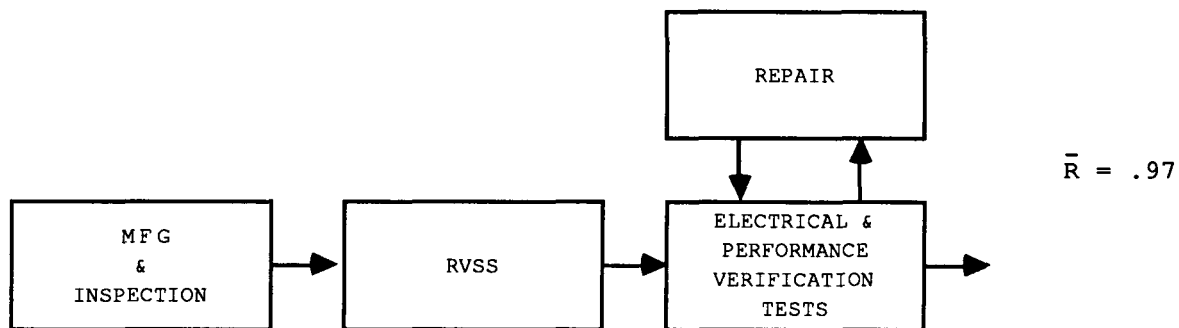


FIGURE 1(a). BASIC RVSS PLAN, STANDARD QUALITY LEVEL

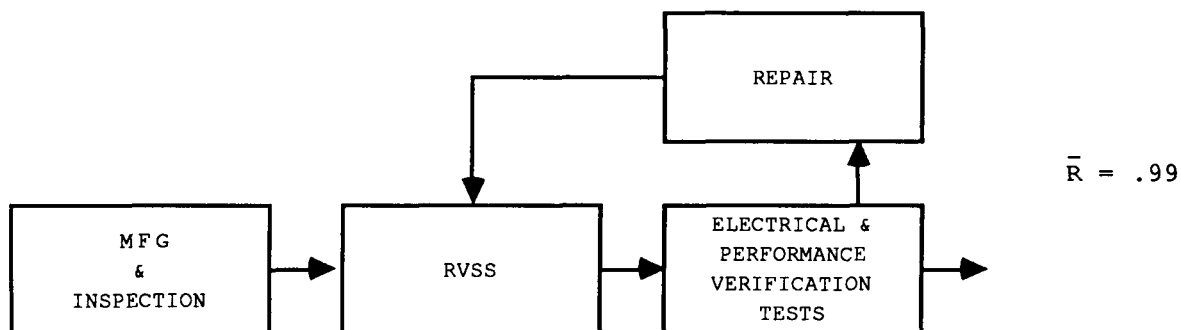


FIGURE 1(b). BASIC RVSS PLAN, HIGH QUALITY LEVEL

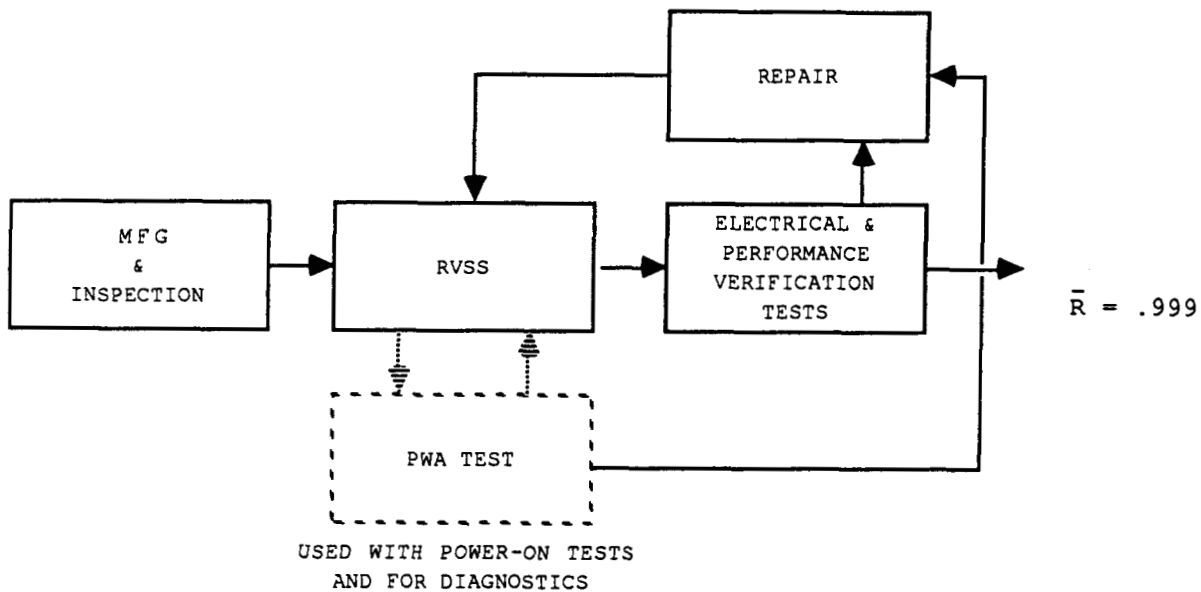


FIGURE 1(c). BASIC RVSS PLAN, VERY HIGH QUALITY LEVEL.

RELATIVE VALUE OF PWA STRESS SCREENING WITH PRE- AND POST-TESTS

Stress screening in general may be categorized in its effectiveness by the illustration in Figure 2. Although the actual numbers may vary from vendor to vendor, there is sufficient data in the field to roughly quantify the relative effectiveness of screening and testing operations. However, the values vary from contractor to contractor. Figure 2 shows that a good pre-screen visual and functional test may surface even more defects than any of the screens. Without the random vibration stress screen, approximately 16% of the defects on a PWA may pass the factory undetected. These ratios vary with each contractor.

OPERATION	DEFECTS FOUND AT OPERATION, RELATIVE FREQUENCY	DEFECTS FILTERED
PRE SS TESTS	<div style="width: 46%;"></div>	46%
TCSS	<div style="width: 34%;"></div>	34%
RVSS	<div style="width: 16%;"></div>	16%
POST SS TESTS	<div style="width: 4%;"></div>	4%
		100%

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FIGURE 2. RELATIVE EFFECTIVENESS OF PWA STRESS SCREEN AND TEST OPERATIONS

RVSS WINDOW AND EXPECTED FALLOUT

The operating stress screen range or window is shown in Figure 3. The objective of a RVSS development program is to determine the upper and lower threshold and produce the optimum level within the window. The upper level may be determined by trial and error or by a combination of analysis and testing. The later approach is preferred since it gives insight to the physical constraints of the PWB, components, and materials involved. The RVSS levels may be obtained by trial and error i.e., determination of the level which produces minimal fallout and the level which causes deleterious effects. The question will inevitably arise, however, "How much higher can we go without over stressing?"

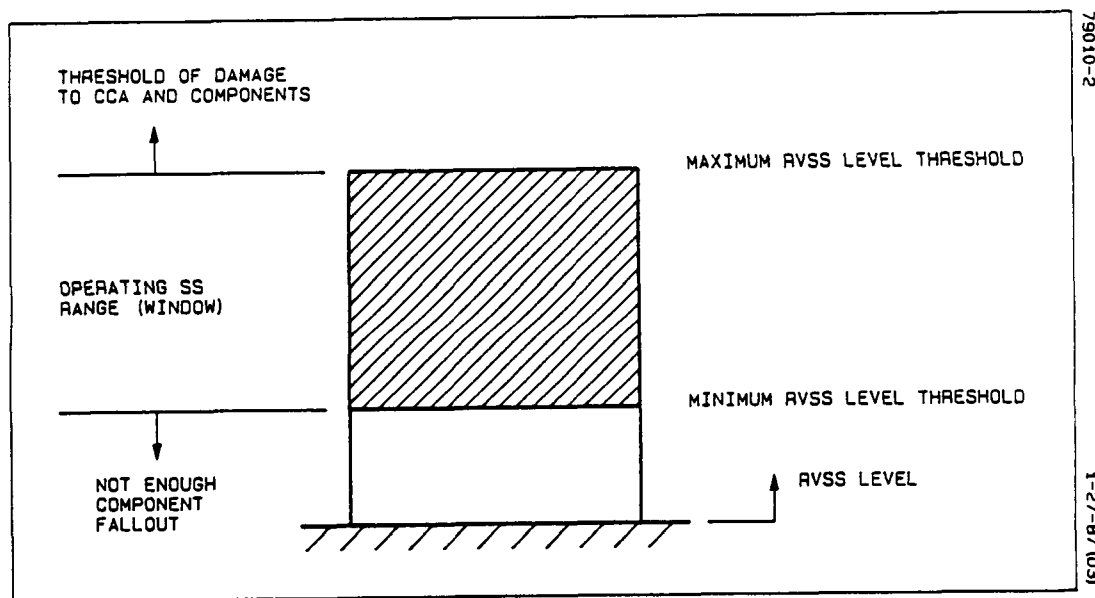


Figure 3 RVSS WINDOW

PWA RVSS INPUT PROFILES AND DYNAMIC CHARACTERISTICS

The input profile of a RVSS is extremely important to the response of the PWA. The response $R(f)$ is proportioned to the transfer function $H(f)$ of the structure multiplied by the input function $I(f)$, as seen in Equation 1.

$$R(f) = |H(f)| I(f) \quad (1)$$

The transfer function $H(f)$ includes all structures between the forcing function (the shaker table) and the unit under test (UUT). This may include a vibration fixture and PWA holder box. Thus, the response on the PWA may include its own dynamic properties plus all the other supporting structure properties. The nature of the random vibration is that it will produce all frequencies in the spectrum range simultaneously, which will stimulate all parts into resonance which have resonances in the input spectrum range. The input level (amplitude) will only

further amplify the motion. A high input level at low frequency resonances may cause excessive bending damage to certain parts, such as the PWB or particular microelectronic devices.

Figure 4(a) shows the NAVMAT profile, which rolls off at the low and high end of a 2000 Hz spectrum. The roll off at the low end helps to diminish the bending stresses on the PWA. The roll off at the high end reduces the high response of small high natural frequency components. The center area between 80 and 350 Hz is the main area of excitation. Most PWA's have natural frequencies in this range. Certain PWA defects, however, may be surfaced with more stimulation at the low frequencies. A profile such as that shown in Figure 4(b) may be used if it does not cause damage to the PWA. If the overall level of .04 is too high and some design damage results, then the overall level may be dropped. There is some evidence that this profile may be a more effective screen at a lower Grms level than the NAVMAT profile.

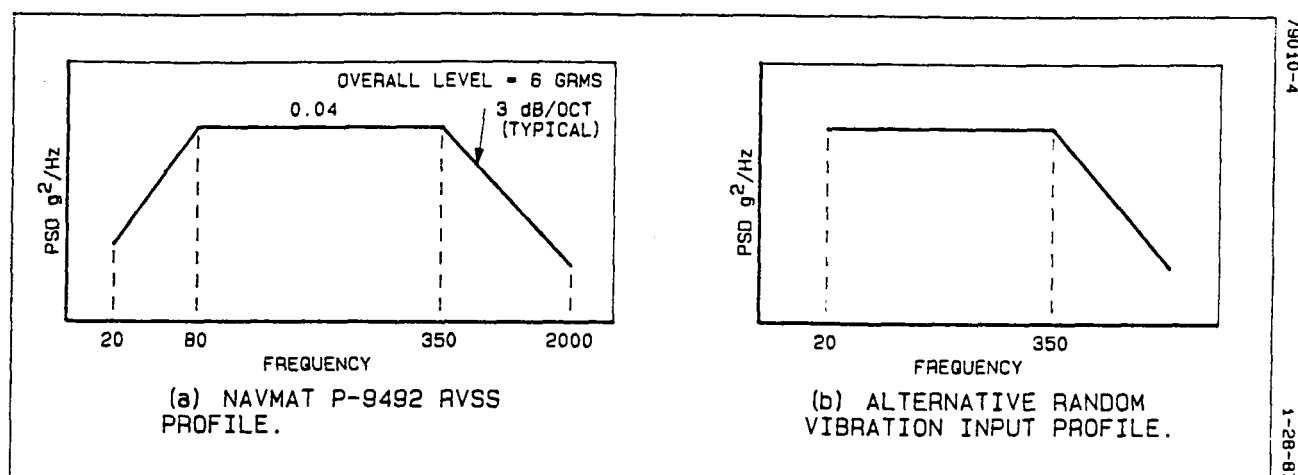


FIGURE 4. INPUT PROFILES

Figure 5 shows a graph of a time history response of a PWA. The time history curve has limited application. However, it does show that there is a good signal voltage and there are no visible signal anomalies. Figure 6(a) is a wide band random vibration process typical of an input profile to the shaker table. Figure 6(b) is a narrow band random vibration process typical of a PWA response to an input from Figure 6(a). The frequency spectrum in Figure 6(b) might be a response to a time history of Figure 5.

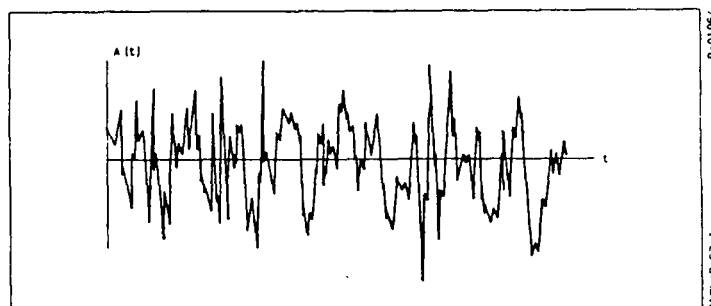


FIGURE 5. RANDOM VIBRATION TIME HISTORY

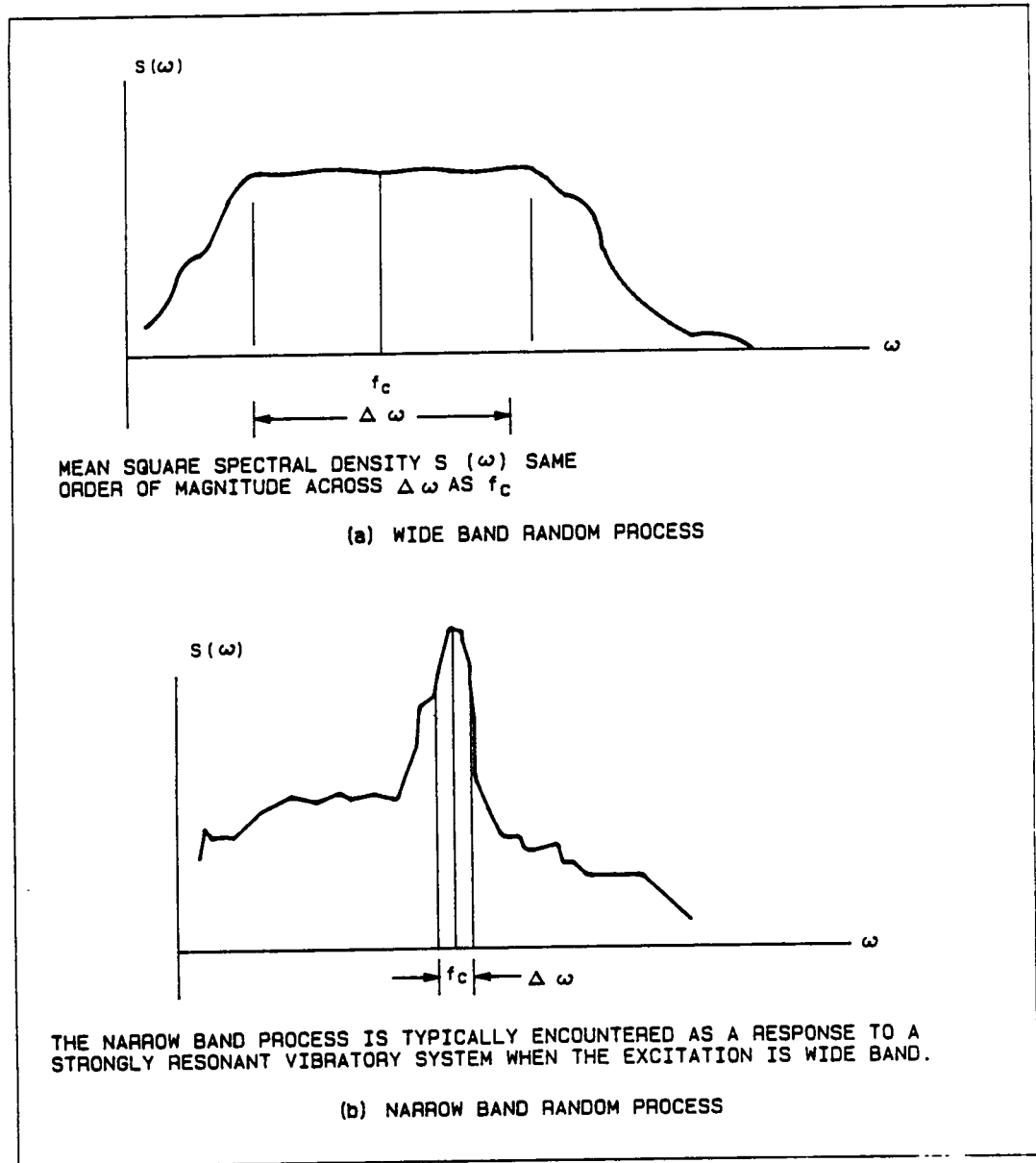
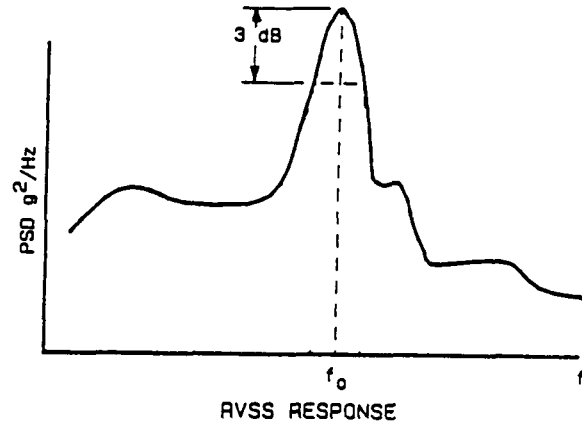
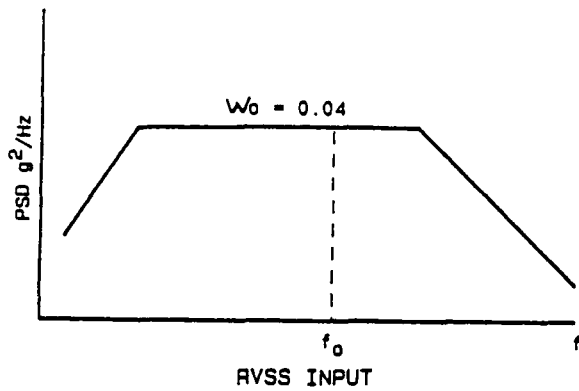


FIGURE 6. RANDOM VIBRATION CURVES

RESPONSE PROFILES TO RANDOM VIBRATION

The RVSS response curve for a given input can be shown as:



From Crandall¹

$$G_r = \sqrt{\frac{\pi f_n Q \omega_o}{2}} \quad (2A)$$

where Q = Magnification of Motion at resonance.

Since $Q \approx \sqrt{f_n}$, at the PWA resonance frequency

$$G_r = \sqrt{\frac{\pi f_n^{1.5} \omega_o}{2}} \quad (2B)$$

EQUIVALENT SINE ACCELERATION TO PRODUCE SAME FATIGUE AS RANDOM VIBRATION

From Harris and Crede² and from $Q \approx \sqrt{f_n}$,

$$G_{RMS} = 1.5 \left(\frac{\pi f_n \omega_o}{2 Q} \right)^{1/2} \quad (3)$$

For peak acceleration, :

$$G_{peak} = 2.7 \left(\frac{f_n \omega_o}{Q} \right)^{1/2} \quad (4)$$

Equations (3) and (4) show the G_{RMS} and G_{PEAK} equivalents for sine acceleration equivalence to random vibration. The sine equivalent G level produces somewhat questionable test results. A sine equivalent did not produce the same results as the RVSS. However, it may be preferred by some companies. Possibly the

¹S.H. Crandall, "Random Vibration in Mechanical Systems", Academic Press, N.Y., 1963

²C.M. Harris and C.E. Crede, "Shock and Vibration Handbook", McGraw Hill, N.Y., 1976

determination of the proper sweep rate plus an appropriate G level would produce the same results.

MAGNIFICATION FACTORS AT RESONANCE, Q

The displacement magnification factor Q is the magnification at resonance; for example, at

$$f/f_n = 1.$$

By definition, the Q depends on the damping ratio,

$$\zeta = C/C_{crit}$$

Figure 7 shows a magnification factor curve for a single degree of freedom system.

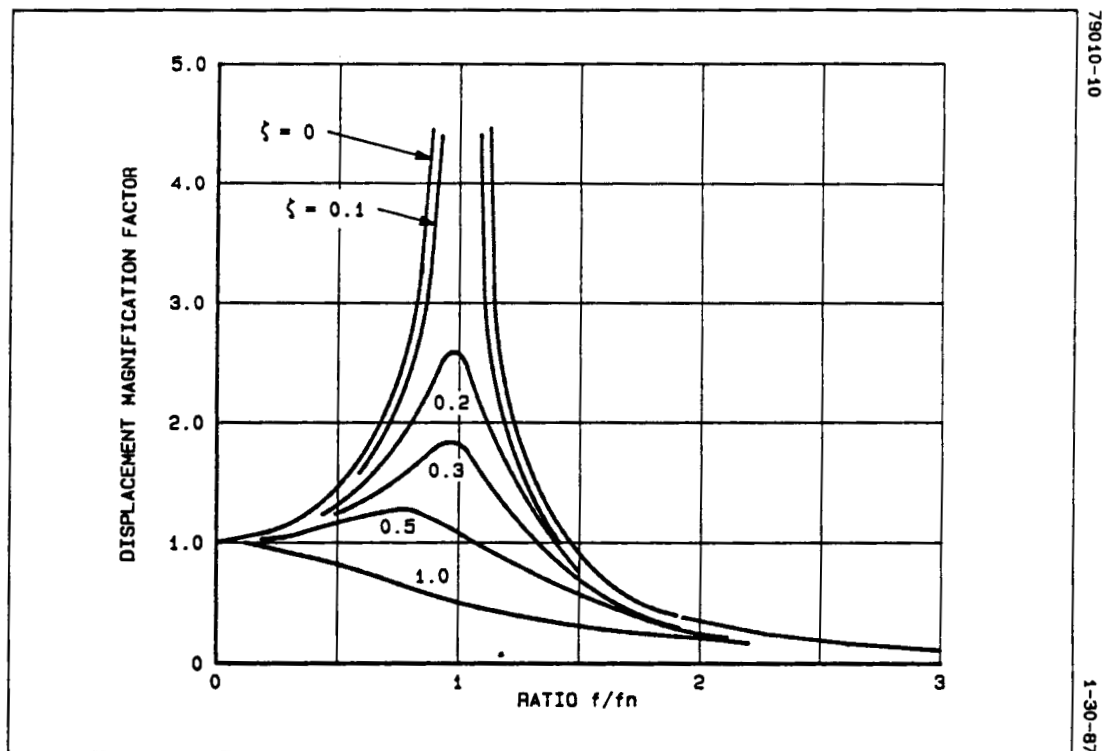


FIGURE 7. MAGNIFICATION FACTOR VERSUS FREQUENCY RATIO

For PWAs, the " ζ " varies with the natural frequency and other factors. However, Q may be estimated as shown in equation (5) if test data is not available.

$$Q = K \sqrt{f_n} \quad (5)$$

Where K is an approximate empirical correction factor for a wide variety of PWAs. A graph of frequency vs.K is shown in Figure 8.

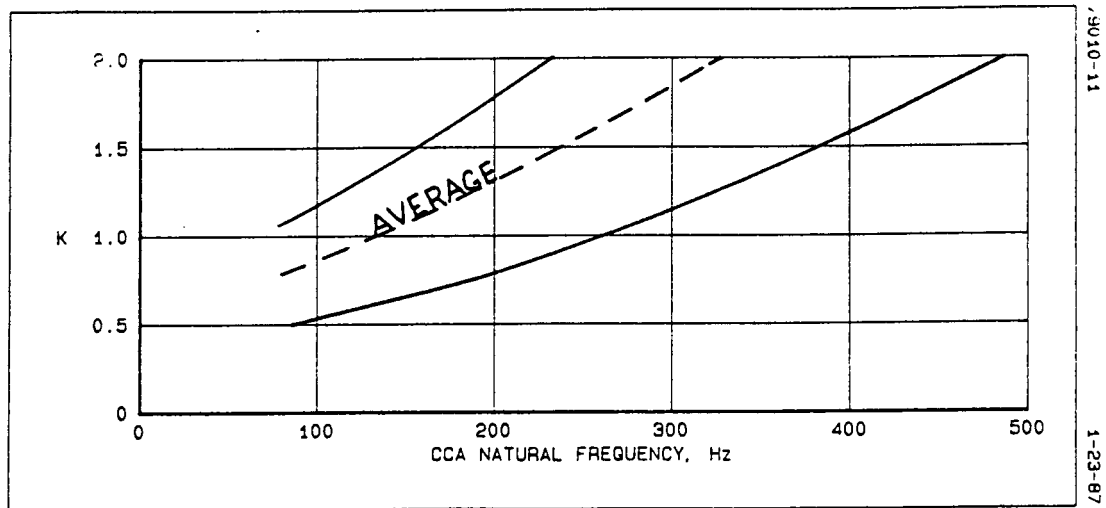


FIGURE 8. CORRECTION MULTIPLICATION FACTOR FOR Q
(MAGNIFICATION AT RESONANCE) FOR PWAs

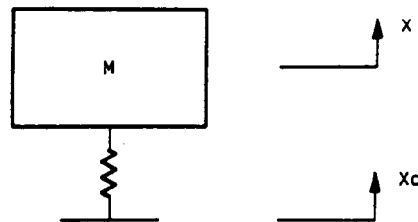
From the standard formula (Equation 6) for determination of the natural frequency for any simple single degree of freedom system,

$$f_n = \frac{3.13}{\sqrt{\delta_o}} \quad (6)$$

or :

$$\delta_o = \left(\frac{3.13}{f_n} \right)^2 \quad (\text{for } 1 \text{ G})$$

For an applied vibration input:



SINGLE DEGREE OF FREEDOM

and $X = X_o Q$ for magnification at resonance. Therefore, for a general PWA Equation(7) may be used as an estimate for determination of displacement.

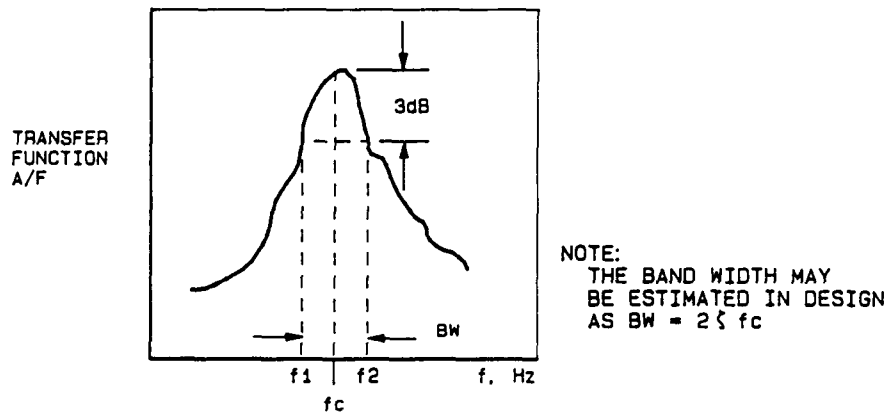
$$\delta = \delta_o Q = \left(\frac{3.13}{f_n} \right)^2 K \sqrt{f_n} = 9.8 f_n^{-1.5} K \quad (7)$$

For example, estimation of PWA displacement at resonance, given $f_n = 120$ Hz, from Figure 8, $K = 0.9$;

$$\delta = 9.8 (120)^{-1.5} 0.9 = \underline{.0067"}$$

OTHER METHODS OF DETERMINING Q (GIVEN TEST DATA)

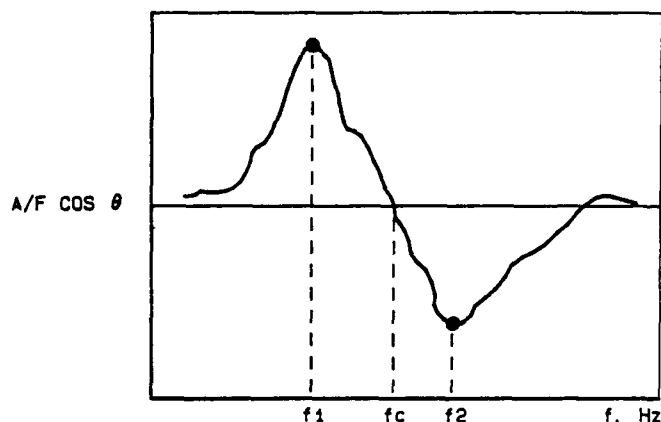
Calculating Q by the "3 dB Down" Method ..This is a quick estimate method but is not quite as accurate as the next two methods.



$$\zeta = \frac{BW}{2 f_n}; Q = \frac{1}{2\zeta}; BW = f_2 - f_1 \quad (8)$$

For our example, $f_n = 160$ and $BW = 18$ Hz, then $Q = 8.9$

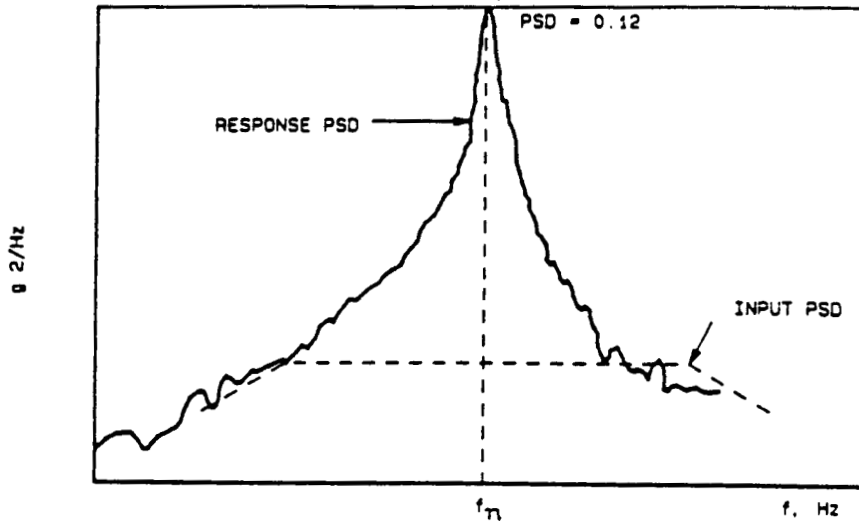
Calculating Q by the Band Width High and Low Frequency Measurement. This is perhaps the most accurate method but requires a Fast Fourier Transform (FFT) analyzer:



$$Q = \frac{1}{2\zeta} = \frac{(f_2/f_1)^2 + 1}{(f_2/f_1)^2 - 1} \quad (9)$$

e.g., $f_n = 160$, $f_2 = 168$, $f_1 = 150$, so $Q = 8.9$.

Calculating Q by the Input/Output Wave-Ratio. This is probably the most popular method of computing Q ; from the figure below,



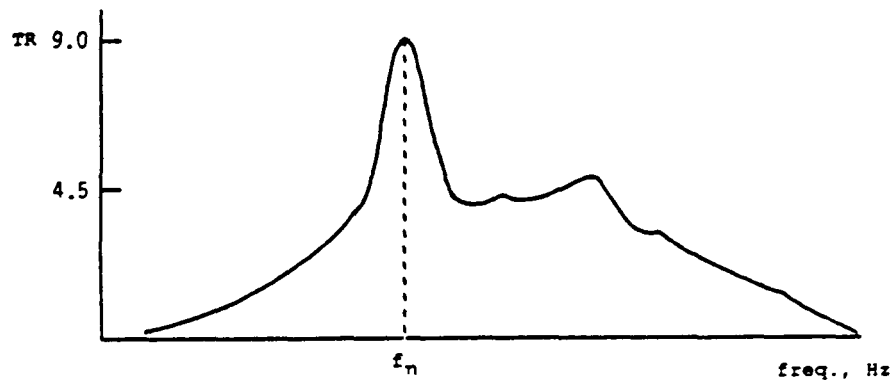
Input PSD = .0015 g^2/Hz at f_n

Response PSD = .012 g^2/Hz at f_n

$$Q = \sqrt{0.12/.0015} = 8.9$$

Calculating Q Directly by FFT, Response/Input Ratio. The magnification ratio factor Q may be determined directly by testing the PWA for response to input ratio across

the entire spectrum by use of a direct transmissibility (TR) analysis by an FFT analyzer. Some FFT analyzers, however, do not have this capability. The figure below is a direct TR analysis which for very low values of ζ is almost equal to Q , the magnification factor.



In the above figure, the $TR = 8.9$; $Q \approx 8.9$

GENERAL MODE SHAPES FOR PWAS

Two PWAs are analyzed using ANSYS finite elements to determine the mode shapes. Actual modal analysis results confirm the mode shapes mainly at the lower frequencies. The PWAs sometimes do not behave as analyzed at higher frequencies, due to the non-linearity of the boundary condition (or edge restraints) and non-linearity of the PWB/component. The mode shapes shown in Figures 8 and 9 are provided for reference and to show that the first mode is usually curved, as assumed in this report. Of course, the amount of distortion is greatly exaggerated for viewing purposes. The actual amount of deflection and distortion diminishes greatly at higher frequencies. Thus the distortion is much smaller at $f = 635$ Hz than at $f = 164$ Hz.

FIGURE 9. MODE SHAPES FOR 7 X 9 PWA

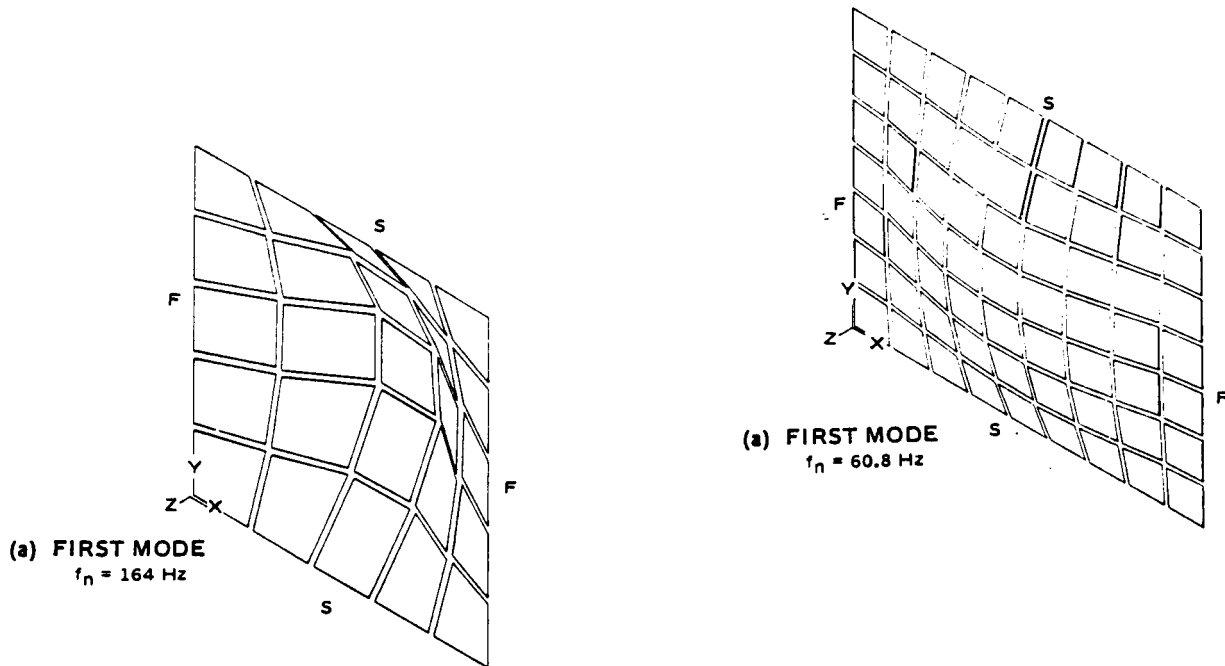


FIGURE 8. MODE SHAPE FOR 5" X 5" PWA

RVSS RESPONSE OVERVIEW

Ideally, the response of a PWA for a random vibration input is dependent on its own structural transfer function. In the real world, the response takes into account the transfer function of all the supporting structures and the shaker itself, as was discussed previously. Thus, the resultant response curve is a conglomeration of PWA dynamic characteristics plus all the support system.

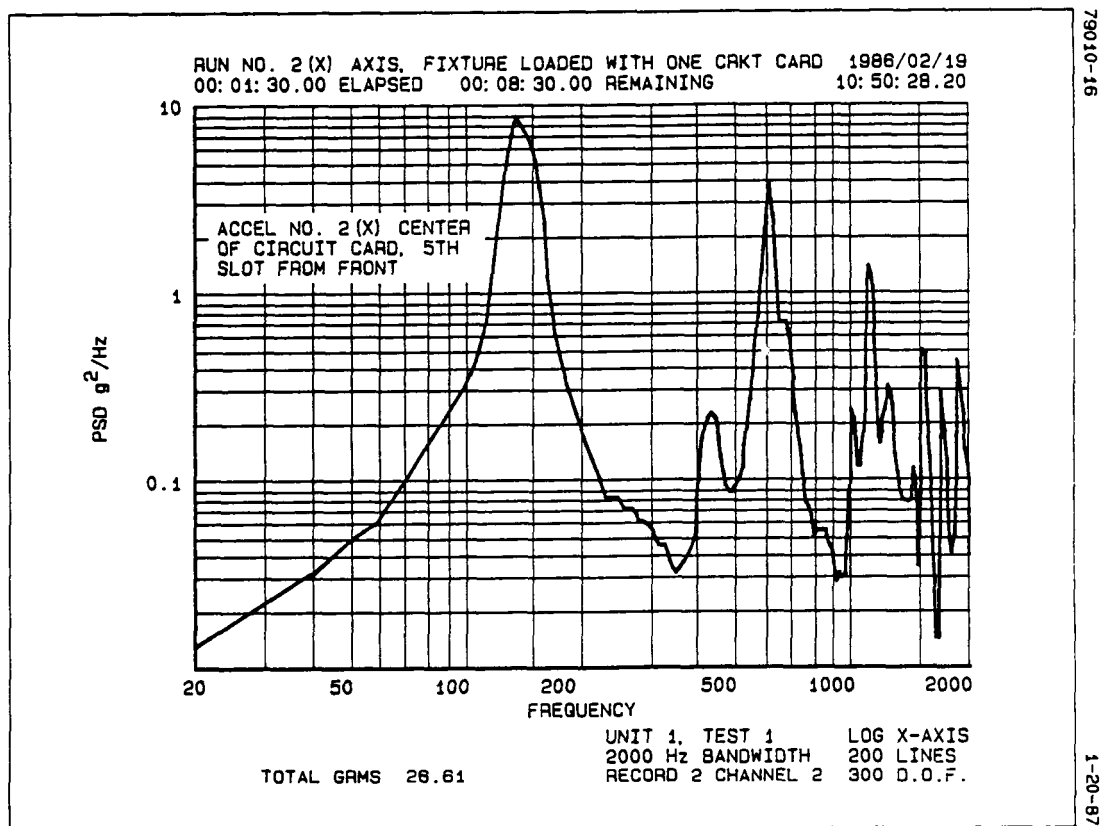


FIGURE 10. RESPONSE OF A TYPICAL PWA

Figure 11 shows a typical PWA signature analysis and response measurement setup. Note that a PWA must have a very light weight accelerometer for measurement at its center, and the input accelerometer may be any weight since it is attached to a rigid fixture.

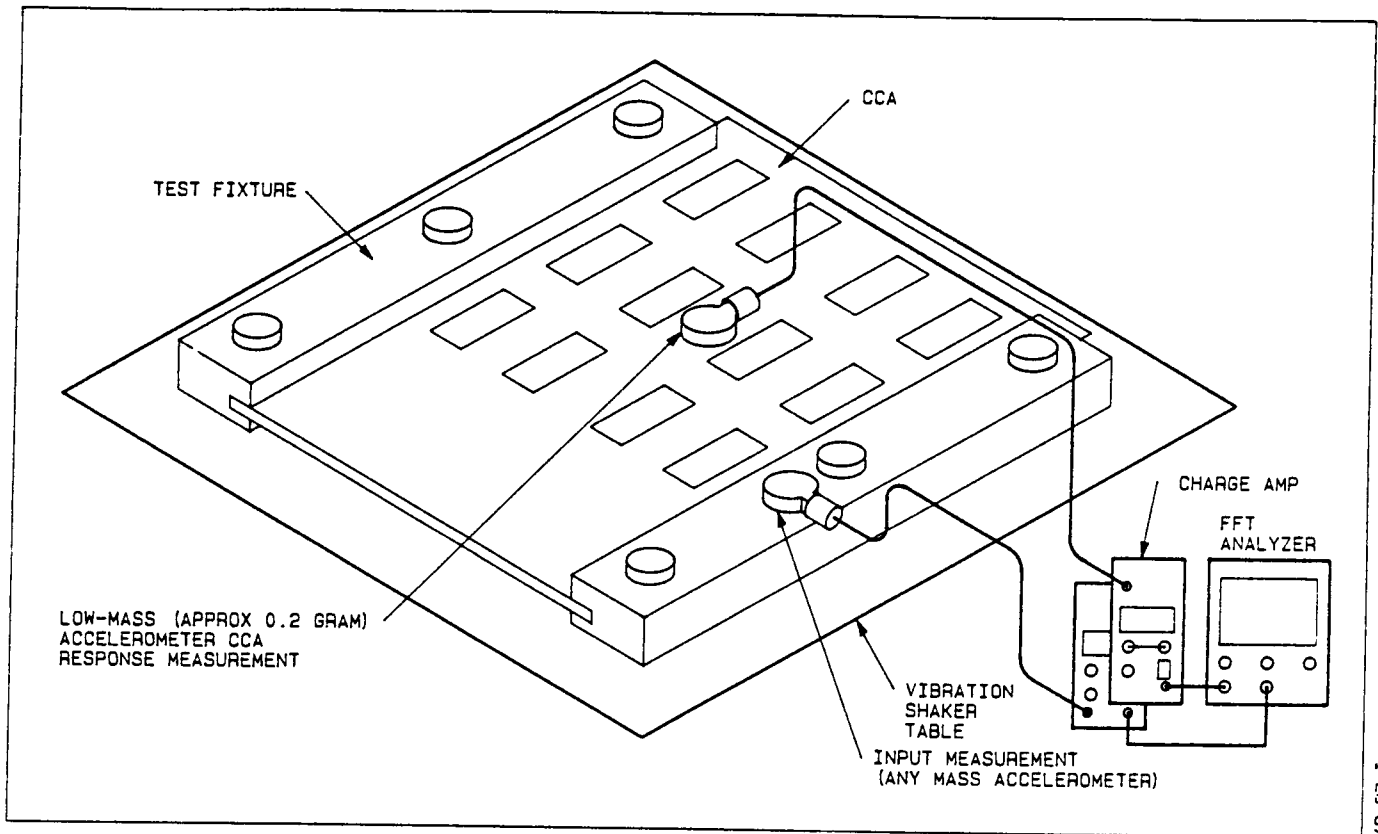


FIGURE 11. PWA TEST SETUP FOR INPUT AND RESPONSE MEASUREMENT

RANDOM VIBRATION RESPONSE ANALYSIS

Refer to the random vibration response shown in Figure 10. In random vibration, the Power Spectral Density (PSD) in G^2/Hz is obtained on a spectrum analyzer by converting real-time data (G's vs.time) into the frequency domain data by a fast fourier transform (FFT). The data may be analyzed more usefully in the frequency domain. Essentially, the spectrum data is divided into constant value filtered bandwidths (BW). The bandwidth, for a specific FFT analyzer (sometimes called the bin width), is obtained by dividing the frequency spectrum range by the number of lines of resolution:

$$BW = \frac{\text{SPECTRUM RANGE}}{\text{NO. OF VERTICAL LINES OF RESOLUTION}} \quad (10)$$

$$\text{e.g., } BW = \frac{2000 - 20}{200} = 9.9 \text{ Hz}$$

The bandwidth is increased by about 50% if the Hanning function is turned on (which is normal). Thus, $BW = 9.9 \times 1.5 = 14.85 \text{ Hz}$

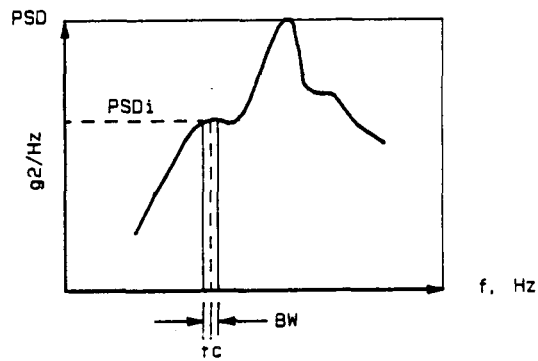
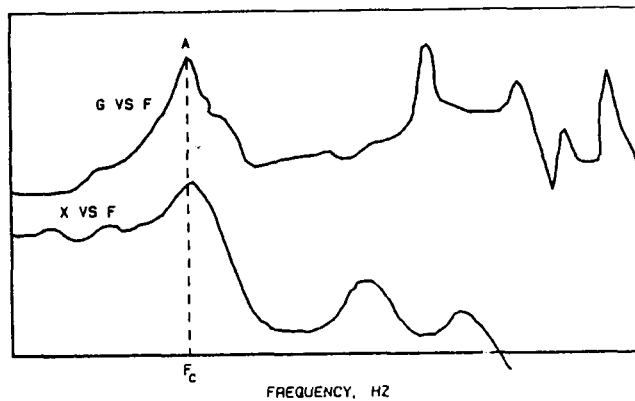


FIGURE 12. DETERMINATION OF G VS. F

The bandwidth varies for each type of analyzer and spectrum range selected. The G acceleration value is computed for each BW, squared, and divided by the constant BW, thus PSD (G^2/Hz). See Figure 12. Although the actual curve is constantly changing with time, a profile is obtained by averaging 64 to 128 times. The G levels of the PSD curve may thus be calculated in reverse by taking a value of PSD corresponding to BW and performing the following calculation:

$$G_{RMS} = \sqrt{PSD_i \times BW} \quad (11)$$

Each "G" is plotted for each center frequency (f_c). The overall effective G_{RMS} is obtained by integrating the entire curve. This is automatically furnished by most analyzers. The overall Grms is essentially the square root of the sum of powers at each frequency. If the G computation is not available on the analyzer, it may be computed manually. It can be seen that the peaks represent a significant part in the overall Grms level. The log-log scale can often be deceiving for judging the bandwidths at each peak.. The results are shown in Figure 13.

FIGURE 13. PSD CURVE CONVERTED TO G_{RMS} AND X_{RMS} VS FREQUENCY

The waveform is now somewhat more subdued due to the square root effect, but the peaks remain evident. Note that the majority of the deflection energy (X_{RMS}) occurs between the lowest frequency and the first mode. The balance of the energy is mostly all velocity energy. If we make an assumption here that the PWA acts like a single degree of freedom (SDOF) system which tests show is nearly the case, the dynamic behavior is much like one spring and one mass at its first natural frequency.

$$\text{from: } a = x\omega^2 \text{ or } G = \frac{x\omega^2}{386}, \text{ where } x = \text{inches} \quad (12)$$

$$\text{Since } \omega = 2 \pi f, \text{ then } x = \frac{386 G}{(2 \pi f)^2} = \frac{9.8 G}{f^2}$$

For PWA's, $x = \delta$.

DETERMINATION OF PWA RELATIVE DEFLECTION FROM TEST DATA

The determination of the response of the PWA relative to its edge (the input) is the goal of the response analysis. The absolute PSD, G , and displacement of the PWA was obtained. The displacement is shown in Figure 14. Subtracting the maximum displacement of the PWA at the critical frequency from the input displacement at the same frequency provides a fair evaluation of the displacement.

$$\delta_{PWA}(\text{relative})_{f_1} = \delta_{ABS_{f_1}} - \delta_{EDGE_{f_1}} \quad (13)$$

Thus in the case shown, the lowest natural frequency of the PWA (first mode) also is the worst case for PWA displacement. As stated before. The analysis requires test data for an actual PWA. However, the first mode response can also be estimated by analytical techniques. The PWA(relative), or δ_{REL} for short, is then compared to the maximum allowable δ_{REL} for PWAs discussed in detail in the following sections of this report. One important aspect of relative motion is that for discrete components (such as axial lead types), the component has no differential motion with the PWA at frequencies lower than the discrete component resonance frequency. At resonance of the discrete component, the component moves out of phase with the PWA and this is where the component wires are stressed. At frequencies above the component natural frequency, the component hardly moves at all, thus nearly no stress. The bending of the PWA, however, can cause lead wire stress.

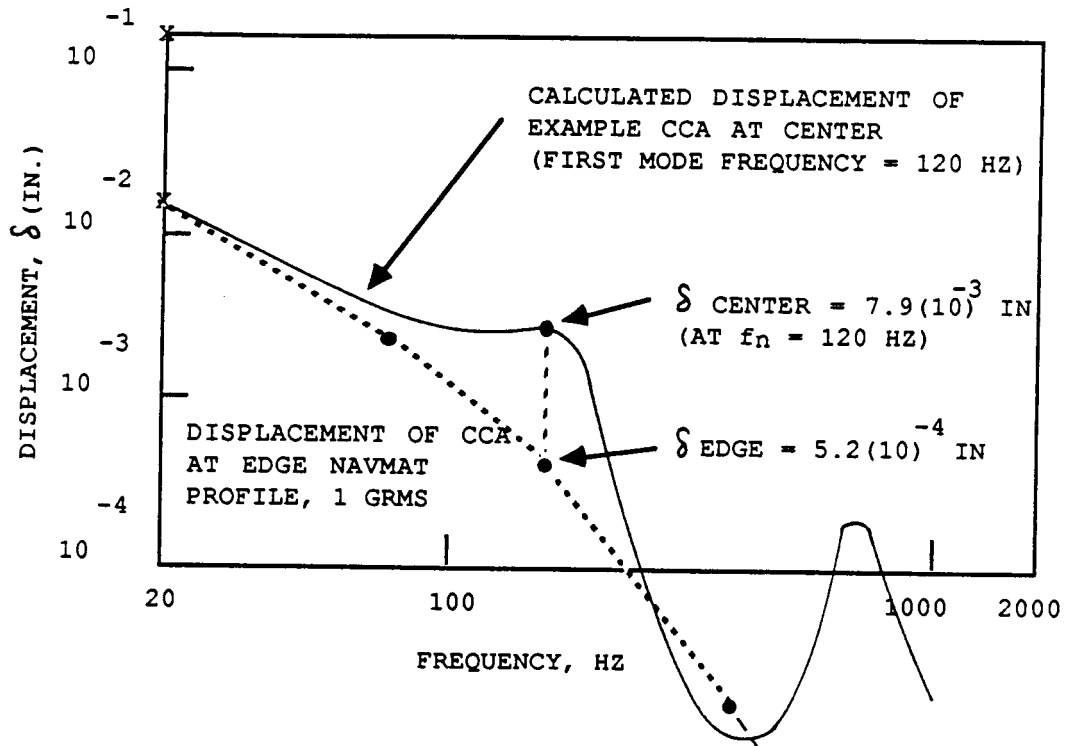


FIGURE 14. DISPLACEMENT OF PWA AT CENTER AND EDGE
(BASED ON EXAMPLE)

From Figure 14,

$$\delta_{REL} = 7.9 \times 10^{-3} - 5.2 \times 10^{-4} = 7.38 \times 10^{-3} \text{ in.}$$

Another method to estimate relative displacement is to apply the Crandall theory, Eqn. (2a):

$$G = \sqrt{\frac{\pi}{2} f_n Q \omega_o}$$

also since,

$$x = \frac{9.8 G}{f_n^2} \text{ and } Q \approx \sqrt{f_n}, \text{ with } K \approx 1$$

then

$$\delta = x = \frac{9.8}{f_n^2} \left(\frac{\pi}{2} f_n f_n^{1/2} \omega_o \right)^{1/2} = 12.3 \left(\frac{\omega_o}{f_n^{5/2}} \right)^{1/2} \quad (14)$$

As an example, from figures 11 and 13;

$$\delta = 12.3 \left(\frac{.04}{120^{5/2}} \right)^{1/2} = 6.19 \times 10^{-3} \text{ in.}$$

$$\delta_{REL} = 6.19 \times 10^{-3} - 5.2 \times 10^{-4} = 5.67 \times 10^{-3} \text{ in.}$$

Thus both methods are reasonably close; however, the first method is the more accurate since the second method is for broadband input, and the NAVMAT curve is not a full broadband across the spectrum. The later method, however, is used in design since the actual curves are usually not available until long after the design stage.

DETERMINATION OF MAXIMUM ALLOWABLE PWA "G" LEVEL FROM TEST DATA

An alternative method to determine whether or not the maximum allowable displacement will be exceeded is to calculate the response G profile, as was done in Figure 15, and to superimpose a line on the curve which represents the maximum permissible G level vs. frequency given a value for maximum relative PWA deflection (δ) at the center.

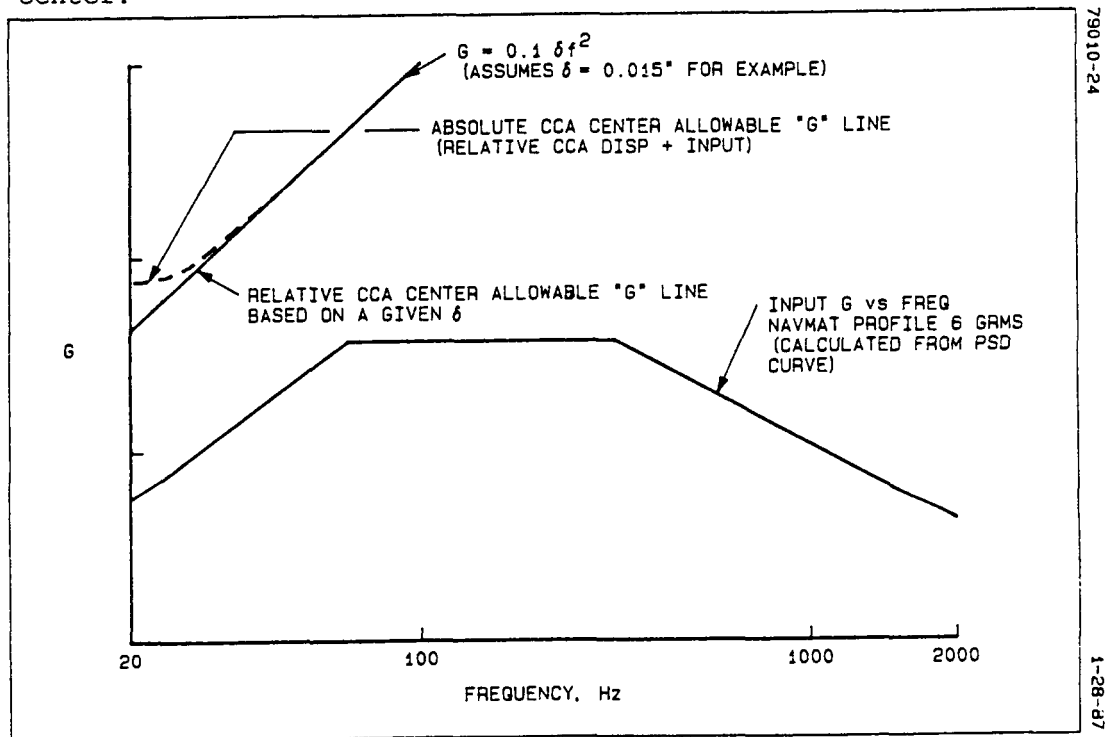


FIGURE 15. RELATIVE AND ABSOLUTE PWA CENTER ALLOWABLE DISPLACEMENT LINE (BASED ON EXAMPLE PWA)

From Figure 15, the absolute and relative PWA center allowable displacement line is determined. The relative displacement is a plot of the equation $G = 0.1 \delta f^2$. The absolute is the addition of this equation plus the input. The

NAVMAT profile (six GRMS input) was used in this example. Note the contribution of the input to the absolute PWA curve. The absolute G line is needed to compare to the absolute G response line in order to compare apples to apples.

The absolute PWA center allowable displacement is plotted on the G vs. frequency response curve in Figure 16.

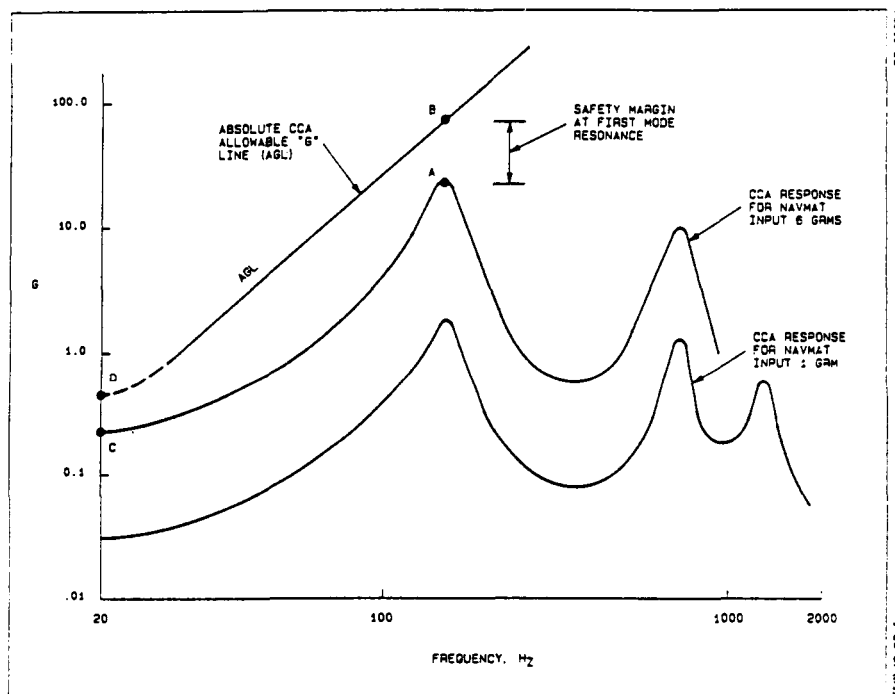


FIGURE 16. MAXIMUM ALLOWABLE PWA "G" INPUT LEVEL

As described above, the absolute allowable G line is obtained by adding the input to the relative allowable G line i.e.,

$$G \text{ ABS. (PWA)} = \text{INPUT } G + G \text{ REL. (PWA)} \text{ each frequency.}$$

Note from Figure 16, the PWA allowable G line (AGL) is established based on a selected maximum allowable deflection (δ) of the PWA at its center. The manually calculated (or analyzer computed display) G vs. frequency response curve is superimposed on the AGL curve and the safety margin obtained. Note that there are two critical areas. The points A & B indicate the safety margin for the first mode resonance, which may be the most critical for causing deleterious effects to the component leads. Points C & D indicate the safety margin at the lowest imposed

frequency. Points A & B may be the most critical and thus both areas should be considered for determining the maximum G response. The higher frequencies do not appear to impose a threat to the PWB or its components since the safety margin is very high. The safety margin should be approximately a factor of 3 to allow for the 3σ statistical variation which could triple the stress 3% of the time. Recall, however, that the AGL shown above was for establishing the maximum deflection or bending of the PWA and does not address lateral or transverse vibration.

The AGL is an approximate method of establishing a limit for the PWA response, but it is not as accurate as the comparison of the PWA actual deflection to the maximum permitted deflection. The response is based on at least 64 to 128 data averages on a spectrum analyzer (which takes only a few seconds). In addition, the bandwidth of the first mode peak contributes to the accuracy. However, as an approximation, tests show it is a reasonably good guideline for most applications.

DETERMINATION OF MAXIMUM ALLOWABLE PSD LEVEL FROM TEST DATA

A maximum allowable PSD level may be obtained by a method similar to the AGL method discussed previously. The premise is as before, namely that a deflection (δ) of the PWA in bending is established from one of the criteria in the following sections. From equation (12),

$$G = 0.1 \delta f^2 \quad (15)$$

This operation establishes the limit of G for a given δ at a given frequency. By squaring the equation and dividing by the filtered bandwidth of Equation (16), we may obtain an allowable PSD limit line (APL) similar to the AGL line.

$$\begin{aligned} G^2/BW &= (0.1 \delta f^2)^2 \\ &= 0.01 \delta^2 f^4/BW \quad g^2/Hz \end{aligned} \quad (16)$$

Equation (16) is the relative APL. The absolute APL is obtained in a similar fashion to the absolute AGL, namely by adding the input PSD curve to the relative APL, i.e.,

$$\overline{APL} = 0.01 \delta^2 f^4/BW + \omega_0 \quad (17)$$

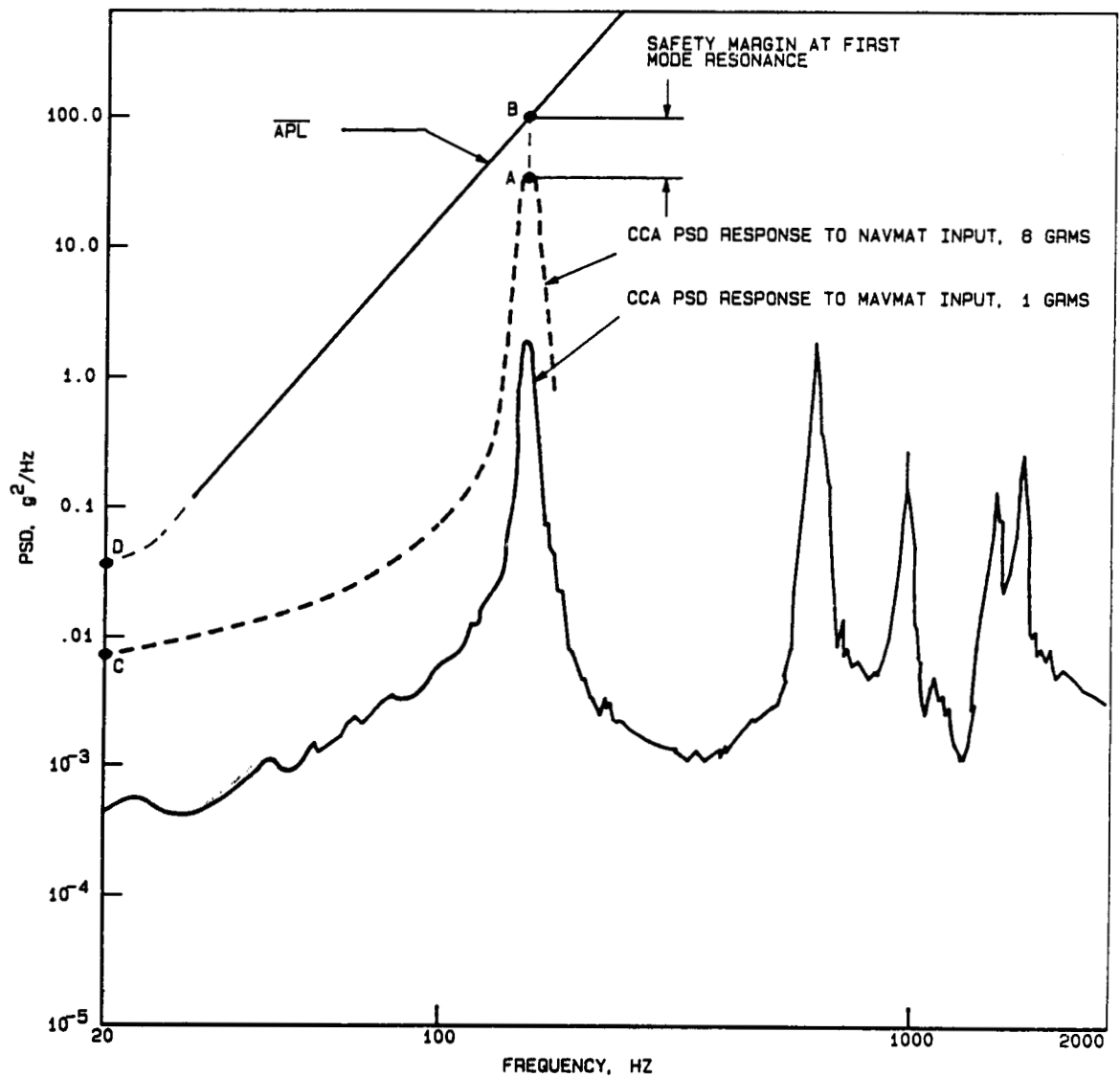


FIGURE 17. MAXIMUM ALLOWABLE PSD INPUT LEVEL

Figure 17 shows the PWA response at two different NAVMAT levels and the allowable PSD limit line. The safety margins are shown similar to the ones in figure 16.

Once again, this method is only approximate and is not preferred to the direct displacement comparison method. The accuracy of this method depends on the same factors mentioned previously. The advantage of this method is that it offers a quick way to check for problems.

PWB BENDING PARAMETERS

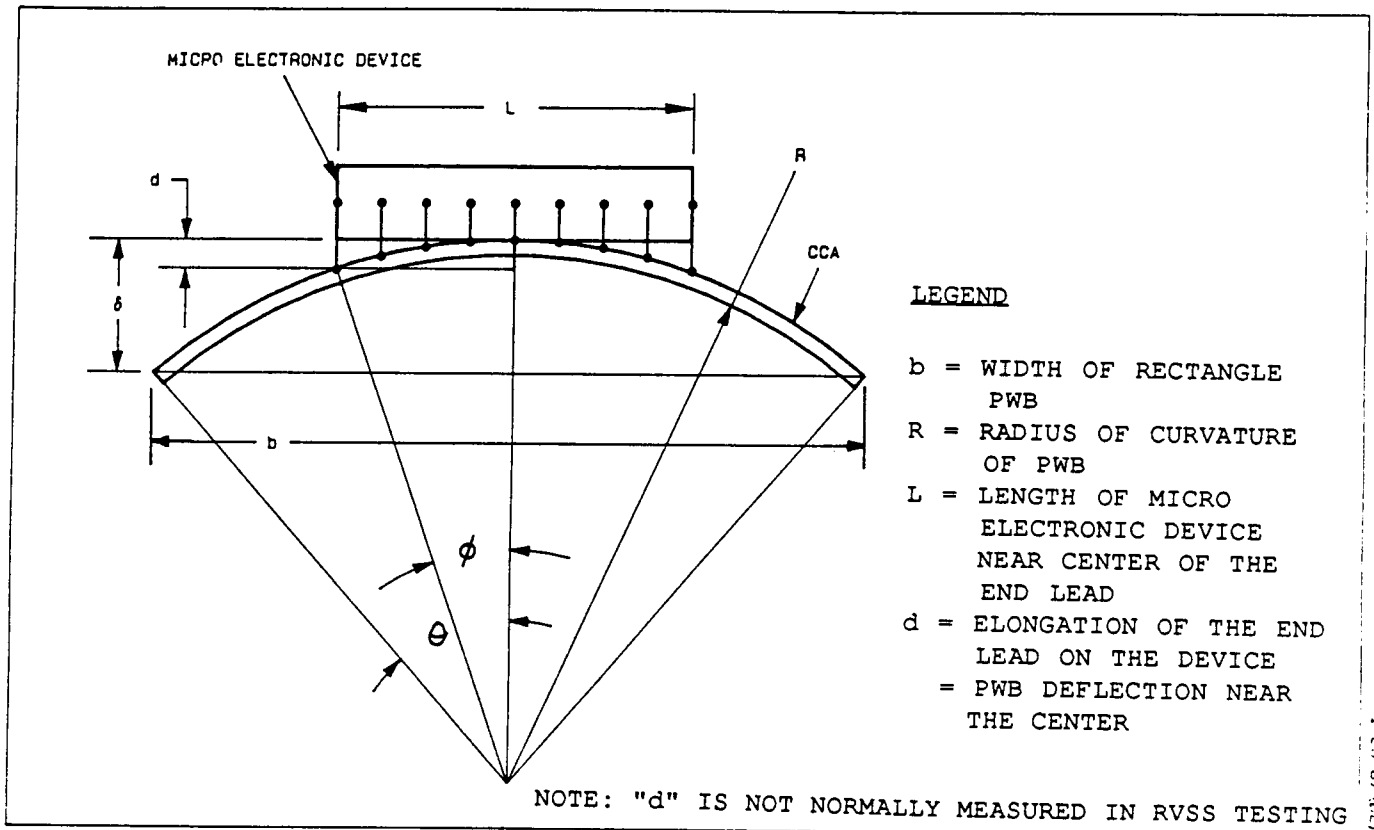


FIGURE 18. PWB DEFLECTION PARAMETERS

From Figure 18,

$$d = \frac{L^2}{8R} \quad \text{or} \quad R = \frac{L^2}{8d} \quad (18)$$

$$\delta = R (1 - \cos \theta) = \frac{R \theta^2}{2} \quad \text{and} \quad \delta = \frac{8 b^2 d}{8 L^2} = \left(\frac{b}{L}\right)^2 d \quad (19)$$

or rearranging

$$d = \delta \left(\frac{L}{b}\right)^2 \quad (20)$$

IC LEADS IN BENDING

The integrated circuit (IC) undergoes stress as result of PWB bending (see Figure 19). The following analysis provides a simple method to determine the stress in the IC leads and the maximum allowable PWA deflection.

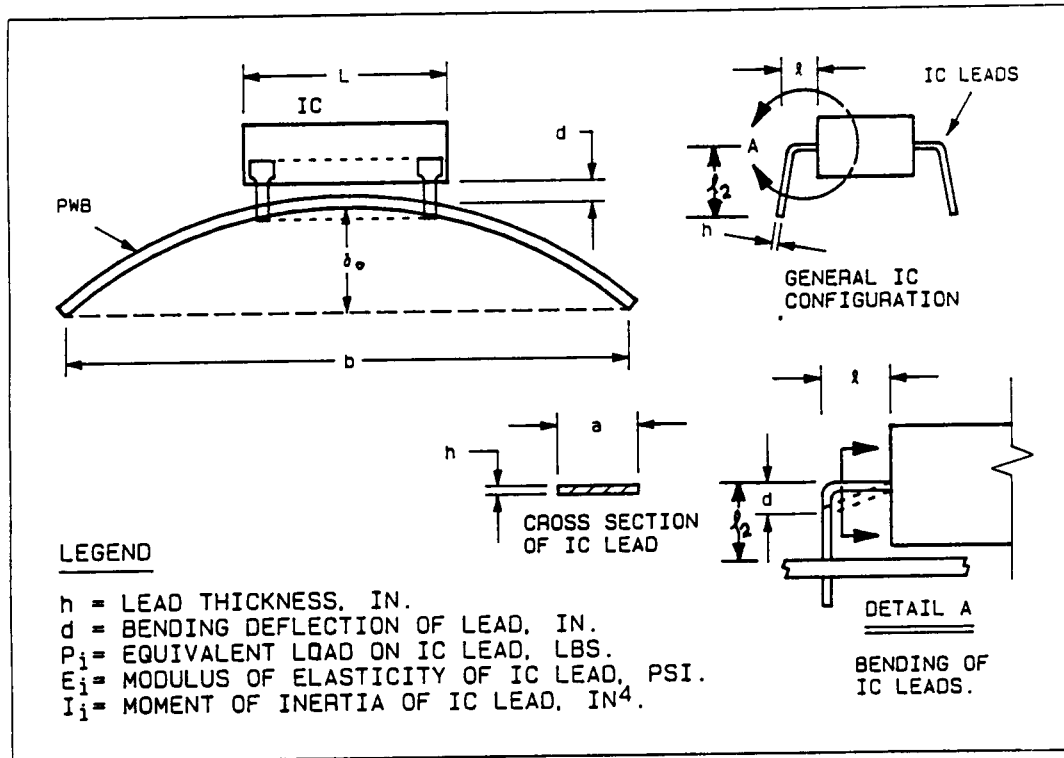
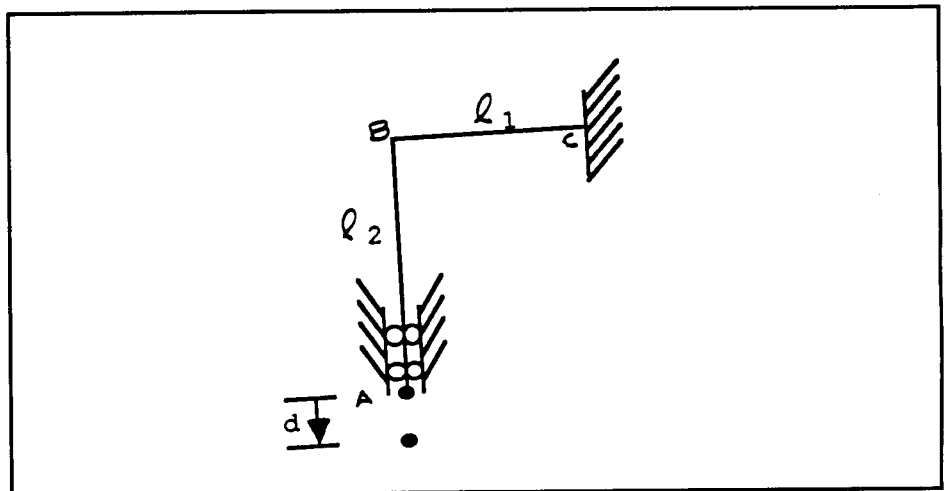


FIGURE 19. GENERAL DIP/IC ATTACHMENT TO PWA

It is desired to determine the stress on the end leads of the DIP for a given PWA displacement.

From the frame structure:

The DIP end leads are deflected a distance "d"
(See Figure 23)



FRAME ANALOGY
FROM THE THEOREM OF CASTIGLIANO

$$d_{TOTAL} \approx \frac{P}{E I} \left[\frac{l_1^3}{3} + l_1^2 l_2 - \frac{l_1^3 (l_1 + 2l_2)}{2 (l_1 + l_2)} - \frac{l_1^2 l_2 (l_1 + 2l_2)}{l_1 + l_2} \right]$$

For most DIPs, $l_2 \approx 4 l_1$

$$d_{TOTAL} = \frac{113}{30} \times \frac{P l_1^3}{E I}$$

The stress on the end lead is:

$$\sigma = \frac{M C}{I} \quad \text{and} \quad M = 9/5 P l_1 \quad \text{and} \quad d = \frac{\delta}{(b/L)^2}$$

$$\sigma = \frac{9}{2} \frac{30}{113} \frac{E}{5} \frac{h}{l_1^2} \frac{\delta}{(b/L)^2} = \frac{.239 E W}{l_1^2} \frac{\delta}{(b/L)^2}$$

or, solving for the PWA deflection in terms of the DIP lead stress,

$$\delta_{MAX} = \left[\frac{4.18 \sigma_{MAX} l_1^2}{E h} \right] \left(\frac{b}{L} \right)^2 \quad (21)$$

The DIP leads may be Kovar or alloy 42. See Table 1 for properties. The stress level is selected on the basis of fatigue normally. The " l_1 " value is .030" for many DIP designs. The "h" value is approx. .010".

TABLE 1. MATERIAL DATA FOR KOVAR AND ALLOY 42

Properties	Kovar	Alloy 42
Ultimate Strength	77,500 psi	70,000 psi
Yield Strength	40 - 59,500 psi	35 - 50,000 psi
E, Modulus of Elasticity	20,000 psi	21,000 psi
Se, Endurance Limit (Fatigue Strength, Estimate Only)	38,750 psi	35,000 psi

A corrective curve (Figure 20) is shown below for modification of equation (21) to allow for the stiffening effect of the component on the PWA. The correction factor is based on some testing of ordinary PWA's and comparison to ANSYS analysis.

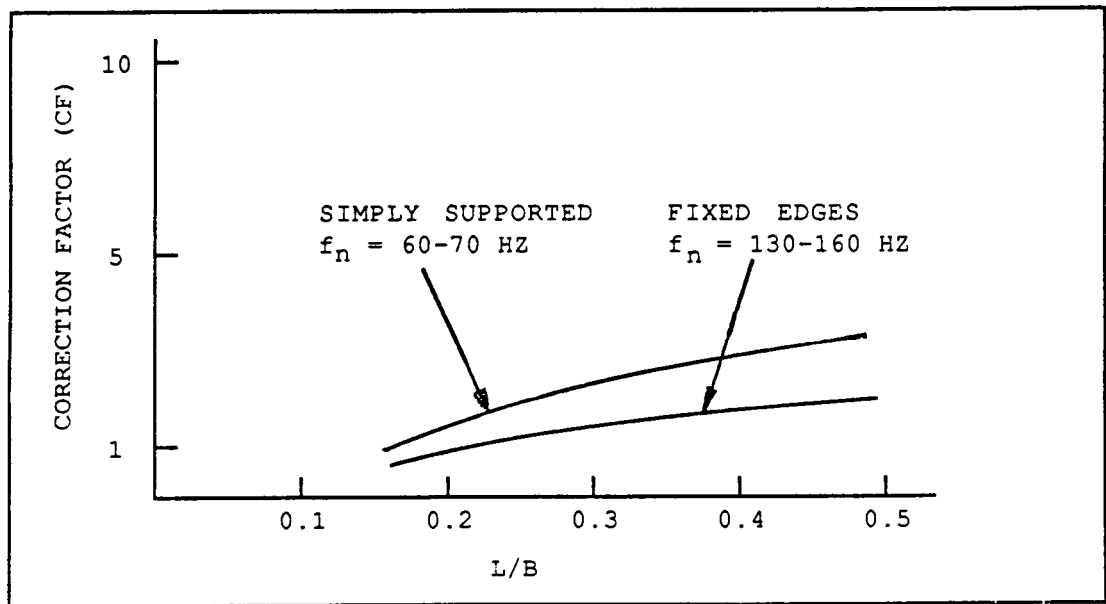


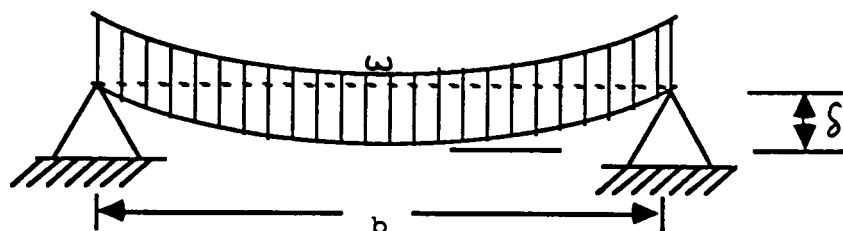
FIGURE 24. CORRECTION FACTOR FOR δ_{MAX} FROM EQUATION (22)

PWB IN BENDING

PWA with Simple Support Edge Restraints A rectangular flat plate may be approximated by a beam whose length is equal to the width of the short side. The higher the ratio of a/b , the more accurate the assumption. The formulas for an ordinary beam with the appropriate boundary conditions may be used. The simplicity of this method lends itself to simple use standard beam equations. The method is illustrated below and is sometimes referred to as the 10% slice method. In this method, the MC/I formulas apply.

The equation for the deflection of a simple supported beam is found in almost all strength of materials text books and structural analysis manuals. The beam length is "b" and the width is assumed to be 10% of the "a" dimension.

As a quick estimate approach, the PWB may be analyzed as a beam with the length of the beam equal to the PWA side "b", and the width equal to 10% of length "a." Thus, the configuration shown below is assumed.



SIMPLY SUPPORTED BEAM

The equation for the deflection of a beam uniformly loaded across the span and simply supported is:

$$\delta = \frac{5 w b^4}{384 E I} \quad \text{where } w = \frac{W}{b}$$

LEGEND

δ = Deflection, in
 b = Short side span of PWA, in
 w = Distributed load, lb/in
 W = Total load, lb
 M = Moment, in-lb
 E = Modulus of Elasticity, lb/in²
 I = Moment of Inertia, in⁴

The moment at the center is:

$$M = \frac{w b^2}{8} = \frac{W}{b} \frac{b^2}{8} = \frac{W b}{8}$$

$$\delta = \frac{5 w b^4}{384 b E I} = \frac{5}{384} \frac{W b^3}{E I} \quad (22)$$

and

$$\delta = \frac{5 M b^2}{48 E I} \quad (23)$$

The bending stress at the center of the PWB is:

$$\sigma = \frac{M C}{I} = \frac{48 E I \delta}{5 b^2} \frac{t}{2 I} = \frac{4.8 E \delta t}{b^2} \quad (24)$$

"I" and "C" may have to be calculated for PWAs with cooling ducts and/or thermal mounting plates; however for a flat plate:

$$\delta_{\text{MAX}} = \left[\frac{0.208 \sigma_{\text{MAX}}}{E} \right] \frac{b^2}{t} \quad (25)$$

TABLE 2. PROPERTIES OF PWB MATERIALS

Material	E, psi	Fatigue Strength, $n = 10^7$ cycles
G-10	2.00×10^6	20,000
Polyimide	2.75×10^6	18,000
Aluminum Oxide	45.0×10^6	12,500

PWA with Fixed Edge Restraints - PWAs are not always restrained as a simple support. A more typical restraint is a fixed end type which is shown in Figure 21.

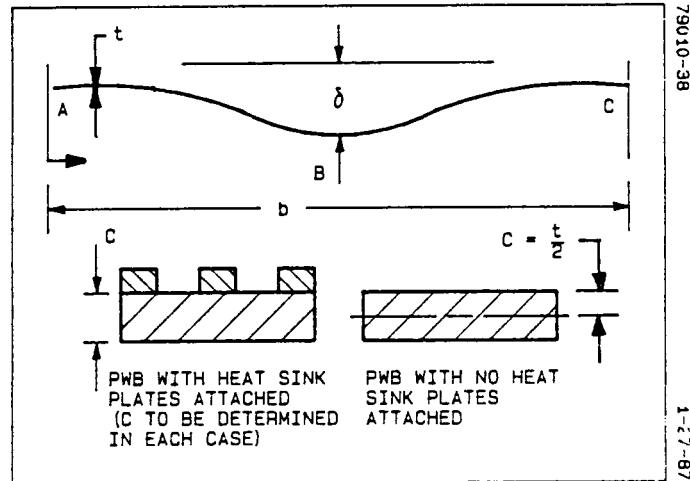


FIGURE 21. FIXED EDGE PWB

In this case, the equation for deflection of a fixed edge beam is:

$$\delta = \frac{W b^3}{384 E I} \quad \text{and} \quad M = \frac{W}{12 b} (6bx - 6x^2 - b^2)$$

$$M_A = .083 Wb \quad M_B = .04 Wb$$

Therefore,

$$\delta = \frac{M_A}{.083 b} \cdot \frac{b^3}{384 E I} = .031 \frac{M_A b^2}{E I}$$

The stress at "A" is:

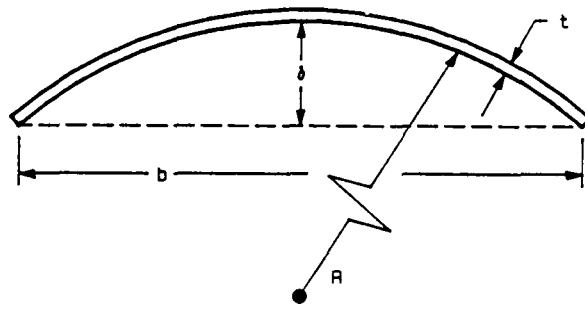
$$\sigma = \frac{MC}{I} = \frac{\delta E I}{.031 b^2} \cdot \frac{t/2}{I} = 16.1 E \delta \frac{t}{b^2}$$

or

$$\delta_{MAX} = \left(\frac{.062 \sigma_{MAX}}{E} \right) \cdot \left(\frac{b^2}{t} \right) \quad (26)$$

ANOTHER APPROACH TO PWB BENDING

A "Simple Support" approach can be analyzed as follows:



BASIC BEAM IN BENDING

For a structure in bending, the moment and radius of curvature is related by:

$$\frac{1}{R} = \frac{M}{EI}$$

$$\frac{8\delta}{b^2} = \frac{M}{EI} \quad \text{where } M = \frac{I\sigma}{t/2}$$

$$\frac{8\delta}{b^2} = \frac{2\sigma I}{tEI} = \frac{2\sigma}{tE}$$

$$\delta = \frac{2\sigma}{tE} \frac{b^2}{t} = \left(\frac{.25 \sigma_{MAX}}{E} \right) \frac{b^2}{t} \quad (27)$$

This compares to Equation (25)

LEADLESS CHIP CARRIERS (LCCs) AND PWB IN BENDING

Leadless chip carriers (LCC's) such as that shown in Figure 22 present a somewhat different problem for determination of the maximum bending and deflection of the PWA in vibration. LCC's may be used on ceramic PWB's which has a material stiffness 16-20 times higher than ordinary G-10 fiber glass epoxy. In the LCC, there are no leads to be stressed. However, the solder tabs at the interconnection act like small springs with different applied forces on them depending on the compliance and curvature of the PWB. The end solder tabs on the LCC are the highest stressed much the same as the DIPs. The soldering quality and workmanship may be particularly important in this type of interconnection since a weak solder joint, cold solder joints, solder voids, etc., directly affect the failure mode. The development of the maximum bending and deflection criteria leads to:

For example,

$$\delta = \left(\frac{\sigma_s l_s}{E_s} \right) \left(\frac{b}{L} \right)^2 \quad (28)$$

where σ_s = Stress level in the solder joint, psi
 l_s = Effective length of the solder joint, in
 E_s = Modulus of Elasticity of Solder, psi

A typical length (l_s) at the solder joint may range from .001" to .003". (See Table 3 for solder strength and other parameters.)

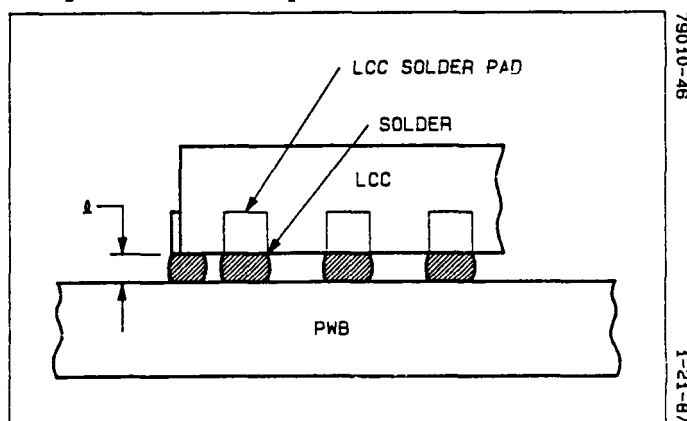


FIGURE 22. LCC SOLDER INTERCONNECTION

TABLE 3. SOLDER STRENGTH VALUES

Sn/Pb/In	U.S. psi	S.S psi	F.S. (estimate) psi	E (estimate) psi x (10) 6
63/37/--	7700	5400	3100	4.6
50/--/50	1720	1630	690	3.4
50/50/--	6200	6200	2500	18.0
--/50/50	4670	2680	1870	2.0

Sn = % tin
Pb = % lead
In = % indium

LEGEND U.S. = Ultimate Strength, psi
S.S. = Shear Strength, psi
F.S. = Fatigue Strength, psi
E = Modulus of Elasticity, psi

From Figure 23, a LCC loaded PWA with power on or in a high temperature environment (or both) may affect the overall fatigue strength of the solder due to local hot spots. Thus temperature, quality and soldering workmanship all directly affect the strength of the LCC/PWA. A combined thermal and vibration stress screen would cause 50% reduction in fatigue strength during a 55°C hot cycle (using 63/37 solder). Thus, the RVSS would have to be reduced in a combined environment.

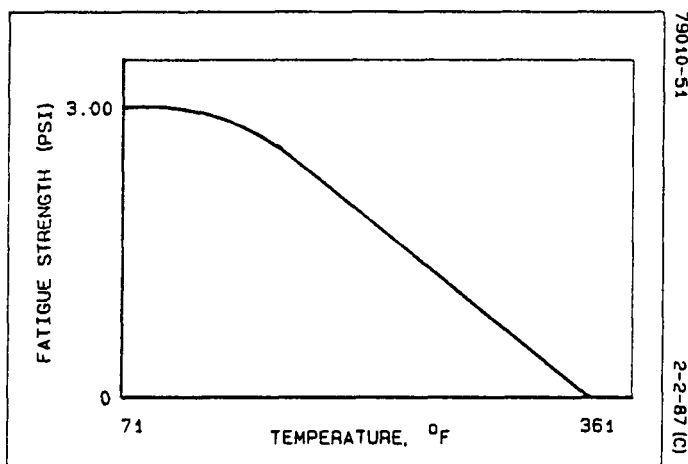


FIGURE 23. FATIGUE STRENGTH VS TEMPERATURE FOR 63/37/0 SOLDER
ESTIMATED VALUES

DETERMINATION OF PWA NATURAL FREQUENCY BY PLATE METHOD

The general equation below is used to solve the general deflection of a PWA with given boundary conditions. The general deflection formula may then be used to determine the natural frequency of the PWA.

$$\delta = \iint A \sin \left(\frac{m \pi x}{a} \right) \sin \left(\frac{n \pi y}{b} \right)$$

Note: δ is in the perpendicular direction to the PWA surface.

The result is of the form,

$$\delta = \delta_0 \sin \left(\frac{\pi x}{a} \right) \sin \left(\frac{\pi y}{b} \right)$$

The solution for natural frequency is derived for any geometry and various boundary condition. The solutions were plotted for a given weight density ($w/ab = .017 \text{ lb/in}^2$), Modulus of Elasticity ($E = 2 \times 10^6 \text{ psi}$), Poisson's ratio ($\mu = .12$), and PWB thickness ($t = .062$ "). The values of natural frequency may thus be obtained directly from the plots. If values other than the above are used,

correction factors may be used and are shown at the end of this section. The basic stiffness factor used in all cases to determine the natural frequency is as follows.

From Meirovitch¹, the stiffness factor of a plate in general is given by "D",

$$D = \frac{E t^3}{12 (1 - \mu^2)} \quad (29)$$

LEGEND E = Modulus of Elasticity, psi
 t = PWA thickness, in
 μ = Poisson's ratio, dimensionless
 $\rho = W/abg = \gamma L/g$ mass density, lb sec²/in³
 W = Total weight of the PWA, lb
 γ = Density, lb/in³
 a = Long length of the PWA, in
 b = Short length of the PWA, in
 g = Acceleration of gravity, 386.4 in/sec²

Two simple cases are calculated for example purposes.

Case 1²

For a PWA with all edges simply supported, the natural frequency is given by:

$$f_n = \frac{\pi}{2} \sqrt{\frac{D}{\rho}} \left(\frac{1}{a^2} + \frac{1}{b^2} \right)$$

for epoxy-glass G-10 $\mu = .12$ and $E = 2 \times 10^6$ psi

$$D = 1.69 \times 10^5 t^3 \frac{\text{lb in}^3}{\text{in}^2} \quad \& \quad \rho = \frac{W}{abg} \quad \& \quad g = 386 \text{ in/sec}^2$$

$$f_n = 1.27 \times 10^4 t^3 a b \left(\frac{1}{a^2} + \frac{1}{b^2} \right)$$

Example a x b = 5" x 5.5", t = .062" PWA and where W = 1.0 lb:

$$f_n = 1.27 \times 10^4 (.062)^3 (5) (5.5) \left(\frac{1}{5.0^2} + \frac{1}{5.5^2} \right) = 71 \text{ Hz}$$

¹L. Meirovitch, "Elements of Vibration," McGraw Hill, N.Y., 1975

²D.S. Steinberg, "Vibration Analysis for Electronic Equipment", J. Wiley & Sons, 1973

Case 2¹

Example plots of natural frequencies for given rectangular plate geometries are provide for various boundary conditions (see Figures 24,25). The weight density, modulus of elasticity, PWB thickness, and Poisson's ratio are assumed for simplication purposes and the assumed values are as stated at the beginning of this section. Other plots for other boundary conditions available on request to the author

To determine natural frequency, find the closest boundary condition case and obtain the natural frequency, f_n .

PLOT CORRECTION FACTORS.

1. WEIGHT DENSITY CHANGE. Calculate $F = W/ab$.

If $W/ab \neq .017 \text{ lb/in}^2$, then multiply the natural frequency f_n by $\sqrt{.017/F}$

e.g., if $F = .025 \text{ lb/in}^2$ and $f_n = 100 \text{ Hz}$

$$f_n \text{ (corrected)} = \sqrt{\frac{.017}{.025}} 100 = 82.5 \text{ Hz}$$

2. MODULUS OF ELASTICITY CHANGE. Determine E .

If $E \neq 2.00 \times 10^6 \text{ lb/in}^2$, then multiply natural frequency f_n by

$$\sqrt{\frac{E}{2.00 \times 10^6}}$$

e.g., if $E = 2.75 \times 10^6$ and $f_n = 100 \text{ Hz}$

$$f_n \text{ (corrected)} = \sqrt{\frac{2.75 \times 10^6}{2.00 \times 10^6}} 100 = 117 \text{ Hz}$$

3. POISSON'S RATIO CHANGE. Any $\mu < .03$ does not significantly change the result and does not require a correction factor.

4. PWB THICKNESS CHANGE. Determine t .

If $t \neq .062$ ", then multiply natural frequency f_n by

$$(t/.062)^{3/2}$$

e.g., if $t = .10$ " and $f_n = 100 \text{ Hz}$

$$f_n \text{ (corrected)} = \left(\frac{.10}{.062} \right)^{3/2} 100 = 200 \text{ Hz}$$

¹Ibid

NATURAL FREQUENCY FOR .062 PWB WITH 4 EDGES SIMPLY SUPPORTED

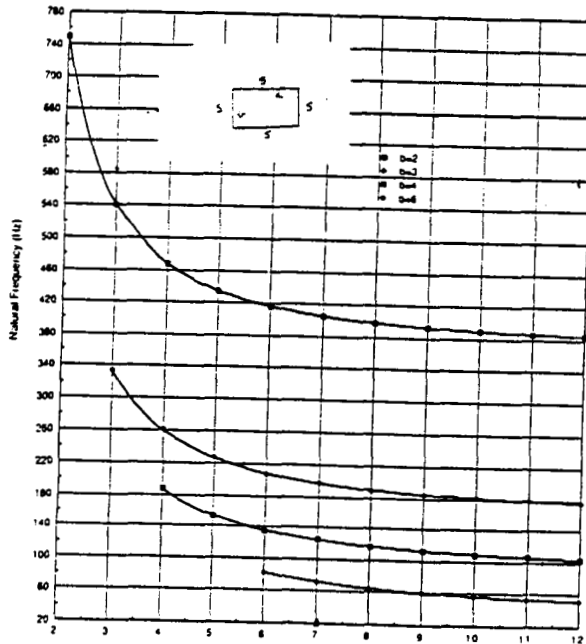


FIGURE 24.

NATURAL FREQUENCY FOR .062 PWB WITH 4 EDGES FIXED

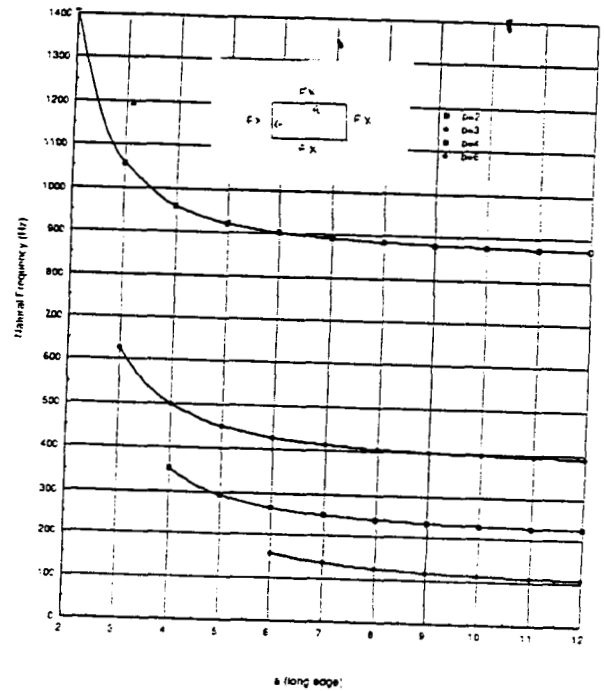


FIGURE 25.

AXIAL LEAD COMPONENTS AND PWB IN BENDING

An axial lead component, such as that shown in Figure 26, is subjected to stress in the leads when a PWA deforms as shown. Equation (31) shows that the stress increases directly as the d/l ratio increases and is also proportional to the span of attachment L . Thus, the components with the highest values for these two parameters are more likely to be damaged in vibration. Equations (32) and (34) provide a guide for the maximum deflection (δ) of the PWA. The values for lead allowable stress and Modulus of Elasticity (E) are obtained from Table 4. It can be seen that steel and nickel wires are very similar structurally. The weight of the component is not a major consideration in this case, but it is important in lateral and transverse vibration. Bonding agents will hold the component down and decrease some of the stress in the lead wires due to vertical motion. There will always be stress in the wires due to PWA bending however.

TABLE 4. LEAD MATERIAL DATA

Lead Material	E 10 ⁶ psi	U.S. psi	S.S. psi	F.S. psi
Steel	30	60,000	30,000	33,000
Nickel	31	52,000	26,000	28,000
Copper (H)	17	48,000	24,000	17,000
Copper (S)	17	33,000	16,500	11,000

LEGEND: E = Modulus of Elasticity
 U.S. = Ultimate Strength, psi
 S.S. = Shear Strength, psi
 F.S. = Fatigue Strength, psi

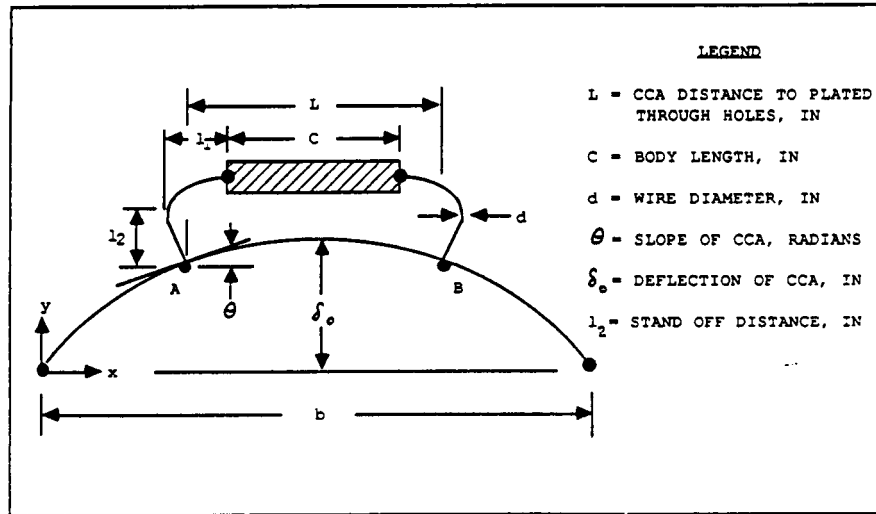


FIGURE 26. DISCRETE AXIAL LEAD COMPONENT IN BENDING

Axial Lead Components and PWB, Simple Support. A typical axial lead component is mounted on a PWA with simple supported edges. The stresses and maximum permitted PWA deflection is analyzed.

From Figure 34:

$$y = \delta_o \sin \frac{\pi x}{b} \quad \text{(General Sine Shape for PWA in Bending)}$$

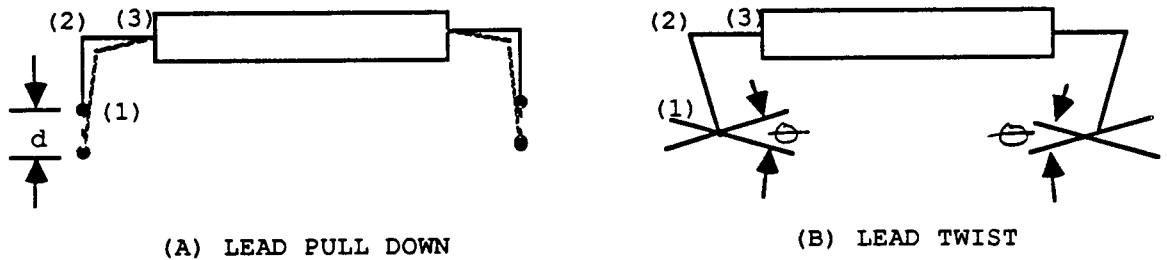
$$\theta = \frac{dy}{dx} = \delta_o \frac{\pi}{b} \cos \frac{\pi x}{b} \quad \text{and} \quad x = \frac{b - L}{2}$$

$$\theta = \delta_o \frac{\pi}{b} \cos \frac{\pi (b - L)}{2b} = \frac{\delta_o \pi}{b} \left(\cos \frac{\pi}{2} - \frac{\pi l}{2b} \right)$$

$$\theta = \frac{\delta_o \pi}{b} \sin \frac{\pi l}{2b} = \frac{\delta_o \pi}{b} \frac{\pi l}{2b}, \text{ since for } \theta \ll 1, \sin \theta = \theta$$

$$\theta = \frac{\delta_o \pi^2 L}{2b^2} \text{ radians} \quad (30)$$

Determination of Stress on Leads. The leads of the component shown in Figure 34 are stressed when a PWA is deflected. The leads are pulled down a distance "d" and twisted through an angle θ as shown below:



The stress in the leads is therefore composed of two parts (bending and twisting - the twisting actually amounts to another form of bending). The stress analysis is therefore performed in two parts, (A) and (B).

Part A

From a derivation similar to equation (21),

where $h = d$ and $d = \text{diameter of lead}$.

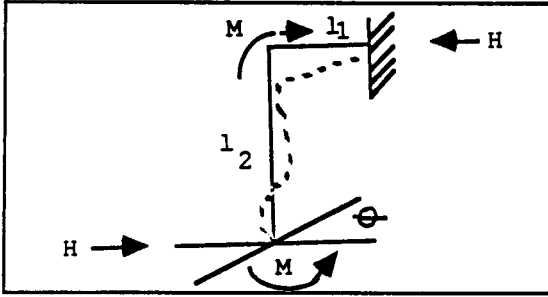
And

$$\delta_{MAX} = (F) \left(\frac{\sigma_{MAX} l_1^2}{E d} \right) (b/L)^2 \quad (31)$$

Where $F \approx 1.048 (l_2/l_1)$ and $0.1 < (l_2/l_1) < 4.0$

Part B

Given the figure shown,



θ = angle of twist of PWA
 M = moment induced in lead

$$\sigma_B = \frac{MC}{I} = \left(\frac{\delta_o \pi^2 L E I}{4 b^2 (l_1 + l_2)} \right) \frac{d/2}{I} = \left(\frac{\pi^2 E d}{8 (l_1 + l_2)} \right) \left(\frac{L}{b^2} \right) \delta_o \quad (32)$$

and

$$\delta_{MAX} = \left(\frac{8 \sigma_{MAX} (l_1 + l_2)}{\pi^2 E d} \right) (b^2/L) \quad (33)$$

NOTE: More information on this subject including axial lead components and methods to determine maximum RVSS input levels may be obtained by contacting the author.